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Complex mode indicator function to find repeated roots or closely coupled modes

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Numerical modal analysis by finite element analysis (FEA) and experimental modal analysis (EMA) with multiple references were used to find and estimate modal parameters of closely coupled modes for a selected structure, a rectangular aluminium plate. The plate is constructed in such a way to have closely coupled modes. The objective was to find closely coupled modes of the plate using the set of frequency response functions (FRFs) that were obtained as a result of the experimental modal analysis. Modal parameter estimation the complex mode indicator function in ME'scopeVES was applied to the plate to find those modes. Normal mode dynamics by FEA was used to determinate their existence before EMA.

Key words: **polyreference method, modal analysis, finite element analysis (FEA), frequency response functions (FRFs), complex mode indicator function (CMIF), closely coupled modes, mode shape, damping.**

Denotations and symbols

N_0	–number of response points,
N_i	–number of excitation points,
N_k	–number of repeated roots,
N_r	–number of dominant modes,
Q_r	–scaling factor for r^{th} mode,
λ_r	– r^{th} system pole,
$j\omega_p$	frequency domain variable,
$\mu_k(j\omega)$	– k^{th} eigenvalue of the normal matrix of FRF matrix,
$\sum_k(j\omega)$	– k^{th} singular value of the FRF matrix,
f	–frequency (Hz)
ζ	–damping (%)
$\{\phi\}_r$	– r^{th} mode shape,
$\{L\}_r$	– r^{th} modal participation factor,
$\{u(j\omega_p)\}_k$	–unscaled mode shape for k^{th} repeated root,
$\{v(j\omega_p)\}_k$	–equivalent mode participation factor for k^{th} repeated root,
$[H(j\omega)]$	–FRF matrix,
$[A_r]$	– r^{th} residue matrix,
$[\Phi]$	–mode shapes matrix,
$[L]$	–modal participation factor matrix,
$[U(j\omega)]$	–left singular matrix,
$[\sum(j\omega)]$	–singular value matrix,
$[V(j\omega)]$	–right singular matrix.

Introduction

SOME test cases in experimental modal analysis require measurements with more than one reference point. Multiple reference testing is required in cases when a structure is very complex (consists of many different parts with different structural properties) or when the structure has more modes with the same or very close natural frequency (repeated roots or closely coupled modes) [1]. The structures with repeated roots or closely coupled modes have been found in test practice very often. One of them is a rectangular plate that is chosen as an example here. In the first test case of experimental modal analysis with a roving hammer, ignoring the existence of closely coupled modes, one referent point at the plate was chosen. Based on a set of frequency response functions two close frequencies were found, but the obtained mode shapes had the same form. Then finite element analysis (FEA) was used to find natural frequencies and mode shapes before another test. It was obvious that two closely coupled modes were found by FEA. Their frequency values were very close while mode shapes were completely different. In the next test case, the set of frequency response functions was found applying experimental modal analysis with two reference points at the plate. Multiple reference modal parameter estimation, the complex mode indicator function (CMIF) was used then to find and to estimate modal parameters of closely coupled modes (frequency, damping and mode shapes).
Finite element analysis of the plate and the second test case with two reference points are described here. Theoretical part, which describes multiple reference modal testing, curve fitting methods and the complex mode indicator function (CMIF) are represented briefly, also.

Multiple reference modal testing

The number of references defines the type of modal testing, which could be single or multiple [2].

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Single reference modal testing is the most common type of modal testing using either a single fixed input or a single fixed output. A roving hammer or a shaker is used as an excitation and a single transducer measures the acceleration, for example. Single input multiple output (SIMO) is the most frequently used single reference modal testing.

When the structure cannot be adequately excited from one reference or one reference cannot excite all modes of interest or when the structure has repeated roots or closely coupled modes, then more than one reference is needed for modal testing. Therefore, two or more fixed inputs or outputs have to be used. This is called multiple reference or MIMO (multiple input multiple output) modal testing. When the inputs are fixed the FRFs are calculated between each of the inputs and multiple outputs and they form multiple columns of the FRF matrix (the inputs are references). Furthermore, when two or more fixed outputs are used, the FRFs are calculated between each output and multiple inputs and they form multiple rows of the FRF matrix (the outputs are references).

Multiple reference modal testing reduces the likelihood of "missing" a mode during the curve fitting process [3]. A structure has repeated roots or closely coupled modes if two or more of its modes have the same or close frequency but different mode shapes. This problem exists with certain symmetrical structures or very complex structures. To be able to find repeated roots or closely coupled modes, the number of rows or columns of the FRFs matrix must be at least equal to the number of modes at the same frequency. That means that the number of reference points must be at least equal to the number of modes at the same frequency. Hence, with two references, two repeated modes can be correctly identified; with three references, three repeated modes can be correctly identified, etc.

Curve fitting methods

After the modal testing, one of the many curve-fitting methods is used to identify modes of the structure. All of them fall into one of the following categories:

- Local SDOF (single degree of freedom)
- Local MDOF (multiple degree of freedom, or multiple mode method)
- Global
- Multi-reference (poly reference)

Local SDOF method is the simplest and multi-reference methods is the most complicated. The difference between SDOF and other methods is that SDOF methods estimate modal parameters one mode at a time and MDOF, Global, and multi-reference methods can simultaneously estimate modal parameters for two or more modes at a time. In addition, there is the difference between local and global methods. Local methods are applied to one FRF at a time. Global and multi-reference methods are applied to an entire set of FRFs at once. Local SDOF can be applied to most FRF data sets with light modal density. Local MDOF methods must be used in cases of high modal density. Global methods have to be used for cases with local modes. When the set of frequency response functions (FRFs) contains repeated roots or closely coupled modes, multiple reference curve fitting has to be used.

Mode indicator functions

Mode indicator functions (MIF) [4] are normally real-valued, frequency domain functions that exhibit local minima or maxima at the natural frequencies of real normal modes. One mode indication function can be plotted for each reference available in the measurement data. The primary mode indication function will exhibit a local minimum or maximum at each of the natural frequencies of the system tested and every successive mode indication function will exhibit a local minimum or maximum at repeated or pseudo-repeated roots of the order of two or more.

There are three different mode indicator methods, the modal peaks function (MPF), complex mode indicator function (CMIF) and multivariate mode indicator function (MMIF). The modal peaks function is calculated by summing together the real parts, imaginary parts or magnitudes of all FRFs. Complex mode indicator function and multivariate mode indication function have entirely different computational algorithms but they give similar results. MMIF indicates the existence of real normal modes and CMIF indicates the existence of real normal or complex modes and the relative magnitude of each mode. The MMIF is based upon finding a force vector that will exit a normal mode at each frequency range of interest, while CMIF is based upon finding the corresponding mode shape and modal participation vectors. The rational fraction polynomial (RFP) curve fitting method is automatically selected when either the CMIF or the MMIF mode indicator is used.

Rational fraction form

The basic task of all multiple mode methods is to estimate the coefficients in a multiple mode analytical expression for the frequency response function [5]. This is done by curve fitting the frequency response function (FRF). There are essentially two different forms of the FRF, which are used for curve fitting. The FRF can be represented in either rational (polynomial) fraction or partial fraction form. In either process, all the modal parameters (frequency, damping and modal coefficient) for all the modes are estimated simultaneously.

The rational fraction form [6] is the ratio of two polynomials:

$$H(j\omega) = \left. \frac{\sum_{k=0}^{m} a_k s^k}{\sum_{k=0}^{n} b_k s^k} \right|_{s=j\omega} \tag{1}$$

The orders of the numerator and denominator polynomials are independent of one another. The equations are linear and the coefficients are identified during the curve fitting process. The equation (1) is the analytical formulation of the FRF data. FRF is the transfer function evaluated along the frequency axis. The denominator polynomial is called the characteristic polynomial of the system. Their roots correspond the poles of the transfer function and are called the roots of the characteristic polynomial. When the characteristic polynomial is zero, the transfer function is infinite. Solutions (roots) for which the numerator polynomial is zero are the values where the transfer function is zero. These values are called the zeros of the transfer function.

Therefore, solving the roots of the numerator and characteristic polynomials, the poles and zeros of the transfer function can be determined. A root finding solution must then be used to determine the modal parameters.

Complex mode indicator function (CMIF)

Modal identification involves estimating the modal parameters of a structural system from the set of FRFs. Modal parameters include the complex-valued modal frequencies, modal vectors and modal mass. Additionally, most current algorithms estimate modal participation vectors and residue vectors as part of the overall process.

The complex mode indicator function [7] appears to be a simple and efficient method for identifying the modes of a complex system. The CMIF identifies modes by showing the physical magnitude of each mode and the damped natural frequency for each root. The CMIF can detect repeated roots and closely coupled modes due to multiple reference data. The CMIF also gives global modal parameters, such as damped natural frequencies, mode shapes and modal participation vectors. The concept of CMIF is developed by performing singular value decomposition (SVD) of the frequency response functions (FRFs) matrix at each spectral line.

In multiple references modal testing the FRF matrix describes the multiple input/multiple output relationship. The FRF matrix of the structure at each spectral line of an N degree-of-freedom system can be expressed as in equation (2). The mass matrix of the structure is assumed to be the identity for simplification.

$$\left[H(j\omega)\right]=\sum_{r=1}^{2N}\frac{\left[A_r\right]}{j\omega-\lambda_r}=\sum_{r=1}^{2N}\frac{Q_r\{\phi\}_r\{L\}_r^H}{j\omega-\lambda_r} \tag{2}$$

where:

N_0 - number of response points,

N_i - number of excitation points,

$\left[H(j\omega)\right]$ - FRF matrix of size N_0 by N_i,

$\left[A_r\right]$ - r^{th} residue matrix of size N_0 by N_i,

$\{\phi\}_r$ - r^{th} mode shape of size N_0 by 1,

$\{L\}_r$ - r^{th} modal participation factor of size N_0 by 1,

Q_r - scaling factor for r^{th} mode,

λ_r - system pole value of r^{th} mode.

Equation (2) in the matrix form is

$$\left[H(j\omega)\right]=\left[\Phi\right]\underbrace{\left[\frac{Q_r}{(j\omega-\lambda_r)}\right]}_{\text{equivalent singular value}}\left[L\right] \tag{3}$$

where:

$\left[\Phi\right]$ - mode shapes matrix of size N_0 by $2N$

$\left[L\right]$ - modal participation factor matrix of size N_i by $2N$

In equation (2), the response of the structure $\left[H(j\omega)\right]$ due to a unit excitation force at a particular frequency ω can be described as a sum, linear combination, of $2N$ residue matrices $\left[A_r\right]$ divided by the distance between modal frequency (system pole) λ_r and the discrete frequency (sampling frequency location in the Laplace domain) $j\omega$. In addition to the equation, the residue matrix is defined as a product of mode shape $\{\phi\}_r$ and modal participation factor $\{L\}_r^H$, weighted by a scaling factor Q_r (for r^{th} mode). The scaling factor can be an indicator of the magnitude of the mode when the mode shape and modal participation factor are scaled to be unitary vectors.

Using singular value decomposition (SVD) [8], any matrix [A] could be decomposed into a product of three matrixes. If these matrixes are multiplied, the matrix [A] can be written in terms of the linearly independent pieces. Also, using SVD the rank of matrix [A] can be determined. There are many different applications for SVD and one of them is that the SDV is the basis of the CMIF. The FRF matrix from several different references can be decomposed using SVD to determine where the roots (or modes) of the system are. The singular value decomposition is applied on the FRF matrix, here. The FRF matrix is decomposed into the product of three matrixes, equation (4), for each frequency. If the number of effective modes is less than or equal to the smaller dimension (number of responses or references) of the FRF matrix then using singular value decomposition two singular vectors are obtained.

$$\left[H(j\omega)\right]=\left[U(j\omega)\right]\left[\sum(j\omega)\right]\left[V(j\omega)\right]^H \tag{4}$$

where:

N_r - number of dominant modes, which are the modes that contribute to the response of the structure at this particular frequency $j\omega$,

$\left[U(j\omega)\right]$ - left singular matrix of size N_0 by N_r (unitary matrix),

$\left[\sum(j\omega)\right]$ - singular value matrix of size N_r by N_r (diagonal matrix),

$\left[V(j\omega)\right]$ - right singular matrix of size N_r by N_i (unitary matrix).

In equation (4), the middle matrix is a diagonal matrix of singular values, which are plotted as the CMIF curves. For r^{th} mode, when the scaling factor Q_r is constant, the smaller the distance between modal and discrete frequency (see equation (3)), the larger the singular value will be. When two different modes are compared, the stronger the mode contribution and larger residue value (see equation (2)), the larger the singular value will be. The corresponding left and right matrixes are singular vectors related to mode shapes and modal participation vectors respectively. The mode shape and modal participation factor are scaled to be unitary vectors (unitary matrixes).

The CMIF is defined as the eigenvalues solved from the normal matrix formed from the FRF matrix at each spectral line. Multiplying the FRF matrix (equation (4)) on the left by its Hermitian matrix the normal matrix is obtained as:

$$\left[H(j\omega)\right]^H\left[H(j\omega)\right]=\left[V(j\omega)\right]\left[\sum^2(j\omega)\right]\left[V(j\omega)\right]^H \tag{5}$$

In addition, the CMIF is equal to the square of the magnitude of the singular value of the FRF matrix:

$$CMIF_k(j\omega)\equiv\mu_k(j\omega)=\left(\sum\nolimits_k(j\omega)\right)^2 \tag{6}$$

where:

$CMIF_k(j\omega)$ - k^{th} CMIF at frequency ω

$\mu_k(j\omega)$ - k^{th} eigenvalue of the normal matrix of FRF matrix at frequency ω,

$\sum_k(j\omega)$ - k^{th} singular value of the FRF matrix at frequency ω.

Left matrix in equation (4) is related to mode shapes. For the k^{th} eigenvalue curve at frequency $j\omega_p$ the unscaled mode shape can be obtained from equation:

$$\{u(j\omega_p)\}_k = [H(j\omega_p)]\{v(j\omega_p)\}_k \, \mu(j\omega_p)_k^{-1} \quad k = 1,2,...,N_k \tag{7}$$

where:

N_k - number of repeated roots detected at frequency $j\omega_p$

$j\omega_p$ - requency of detected peaks that is the approximate damped natural frequency of r^{th} mode

$\{u(j\omega_p)\}_k$ - unscaled mode shape for k^{th} repeated root at $j\omega_p$

$\{v(j\omega_p)\}_k$ - equivalent mode participation factor for k^{th} repeated root at $j\omega_p$

The mode shapes do not change much around each peak. Several adjacent spectral lines from the FRF matrix can be used simultaneously for the better estimation of mode shapes.

Right matrix in equation (4) is related to modal participation factors. A matrix of modal participation factors indicates how strongly each mode "participates in" the FRFs for each reference, i.e. how well each modal vector is excited from each of the reference locations.

In practice, using ME'scopeVES [9] [10], modal parameter estimation is done in several steps:

1. Determine the ***number of modes*** in a frequency band.
2. Estimate modal ***frequency and damping*** for the modes in the frequency band.
3. Estimate modal ***residues*** for the modes with frequency and damping estimates in the band.
4. ***Save*** the mode shapes into a shape table file.

In the first step of the CMIF, the eigenvalues of the normal matrix are plotted as the CMIF curves on a logarithmic magnitude scale as a function of frequency. There will be as many curves as the number of references. If there are two references, there will be two CMIF curves, also. The CMIF curves are displayed together and each of them has a different colour. Also, all of the peaks above the threshold line are displayed as red dots. An automatic peak detector is used to identify the existence of modes. The eigenvector corresponding to the detected peak is equivalent to the modal participation factor. The modal participation factors are obtained in the first step and then are used as a weighting function during the next three steps. They are kept in memory with the CMIFs but are not displayed. The peaks detected in the CMIF plot indicate the existence of modes of vibration, the location on the frequency axis that is nearest to the pole. It must be noted that not all peaks in the CMIF indicate modes. Errors such as noise, leakage, nonlinearity and a cross eigenvalue effect can also make a peak. For example, the leakage error could be minimized if the several spectral lines of data were included in the singular value decomposition calculation. The cross eigenvalue effect appears when the contribution of two modes is approximately equal at a specific frequency. Then two eigenvalue curves cross each other at that frequency.

In the second step, the RFP method performs multiple reference curve fitting using the modal participation factors to estimate modal frequency and damping for the previously detected peaks. The frequency is the estimated damped natural frequency and is the frequency at which the maximum magnitude of the singular value occurs. Finally, the frequencies and damping of all modes are listed in the spreadsheet.

Once the modal frequencies and damping are estimated, modal residues, (magnitudes and phases) can be estimated in the third step, also using the modal participation factors.

In the fourth step when the curve fitting is completed, mode shapes can be saved according to the relative strengths of the modal participation factors for each reference and the largest participation factor is automatically chosen for each mode. Mode shapes are saved in the shape table file.

Finally, the CMIF have some advantages and disadvantages (limitations) that are useful to know. Some of the advantages of the CMIF are:

- The CMIF identifies the number of modes with the existence of repeated roots or closely coupled modes of the system before modal parameter estimation is applied.
- The eigenvalues can be used as a weighting function.
- The CMIF may be used to determine the optimum number of references necessary to identify all the modes in a frequency band.
- The CMIF minimizes the requirement for user judgment and experience.
- When the contribution of noise is large, the CMIF ignores it by using the singular value decomposition technique and when it is necessary to reduce the effects of errors such as leakage the CMIF uses data over several spectral lines in the singular value decomposition.
- The CMIF is good for Spatial Sine Testing because of the uneven frequency spacing data.

Some limitations are:

- Multiple reference FRFs information is needed for the CMIF calculation. Namely, the number of references must be larger than or equal to the number of dominant modes at each spectral line.
- Frequency resolution limits the accuracy of the modal parameters.
- The knowledge of a reduced mass matrix is needed for a more accurate CMIF calculation.
- Second stage procedure is needed for scaled mode shapes and more accurate pole estimation.

Plate model

One plate of aluminium, whose dimensions are 482.6x177.8 mm and thickness 4 mm, is chosen for the analysis. There are two channels at shorter sides of the plate. The dimensions of those channels are 15.5x7.5 mm. The plate geometry is shown in Fig.1. Plate mass is 0.92 kg.

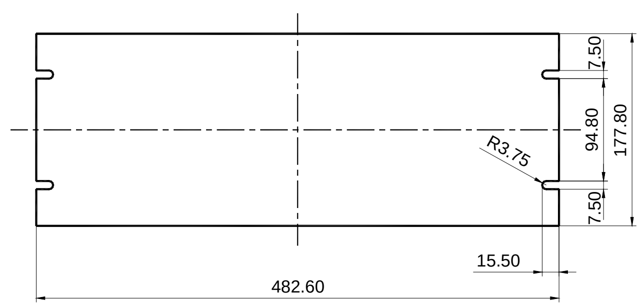

Figure 1. Plate geometry

Finite element analysis (FEA)

Finite element model for the plate

Software I-DEAS was used for numerical modal analysis. Finite element model was generated on the upper surface of the plate. The free thin shell mesh was chosen with element thickness of 4 mm. An accelerometer (which should be used in experimental modal analysis) of 8 g mass (the mass of the glue was ignored) with its base and the part of the cable lay on the plate. Since the mass was big enough to influence the value of natural frequencies, it was modelled as the nonstructural mass. Two accelerometers were planned to be used for the experiment and were placed in the corners of the shorter side of the plate. At those places, the elements of finite element model were 4mm thick and had nonstructural mass of 99.5 kg/m^2 per area. Boundary condition set consisted of all four sides of the plate free (free-free condition). The plate is made of aluminium and its mass with nonstructural mass of two accelerometers is 0.936 kg. The finite element model of the plate had 1016 elements and 1093 nodes and it is shown in Fig.2.

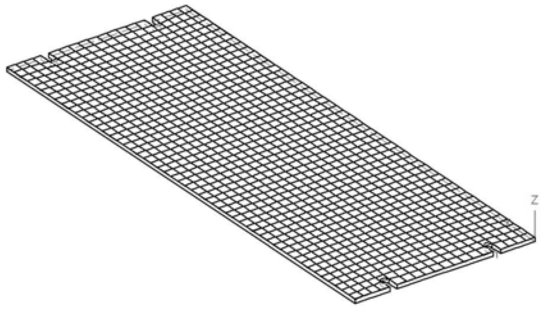

Figure 2. Finite element model

Results of the FEA

Normal mode dynamic analysis by Lanczos' method was applied to the finite element model of the plate. As a result, the analysis gave nine modal frequencies different from zero and nine mode shapes. Because the plate was free, the first six modes were modes of the rigid body and their frequencies were zero. Mode shapes were inertial normalized. Table 1 contains frequency results and it is obvious that the frequencies of the sixth mode (496.11 Hz), the third bending, and the seventh mode (501.61 Hz), the third torsion are very close. The estimation demonstrated that those modes are at very close frequencies so it could be expected to have those closed values of frequencies or for e.g. repeated roots at the experiment. Because of that, polyreference method has to be used to find and estimate modal parameters of closely coupled modes.

Table 1. Results of the FEA

	Description	f (Hz)
1.	Frequency of the first longitudinal bending	89.72
2.	Frequency of the first longitudinal torsion	143.22
3.	Frequency of the second longitudinal bending	250.85
4.	Frequency of the second longitudinal torsion	303.35
5.	Frequency of the third longitudinal bending	496.11
6.	Frequency of the third longitudinal torsion	501.61
7.	Frequency of the first transversal bending	655.90
8.	Frequency of the second transversal bending	729.72
9.	Frequency of the fourth longitudinal torsion	758.96

Mode shapes for two closely coupled modes, the third bending and the third torsion are shown in Figures 3 and 4.

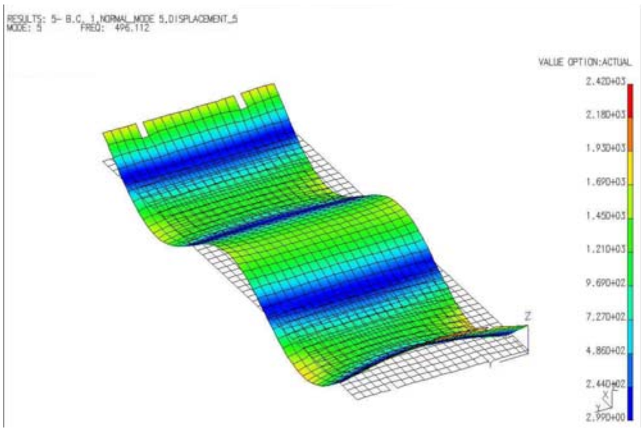

Figure 3. The third longitudinal bending

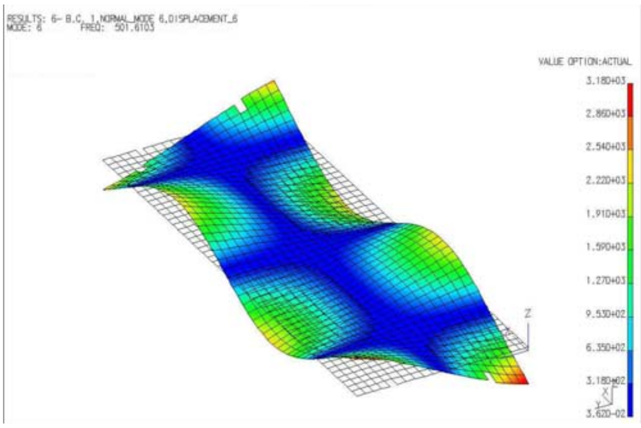

Figure 4. The third longitudinal torsion

Experimental modal analysis (EMA)

Measurement and testing equipment

Testing was carried out with measurement equipment of Laboratory for the experimental modal analysis from the Military Technical Institute. Hammer B&K 2302-10 was used for impulse and accelometers B&K 4507B1 measured the response. The signals were recorded and analyzed by system B&K PULSE 3560D and software PULSE LabShop.

In the post-processing using ME'scopeVES software, one of the mode indicator methods -complex mode indicator function (CMIF) is used to help identify the number of modes in a band. As it was said, ME'scopeVES has a choice between three different mode indicator methods: the modal peaks function (MPF), complex mode indicator function (CMIF), or multivariate mode indicator function (MMIF).

Model for the experiment

Because of the symmetry of its geometry, a square plate will often have repeated roots or closely coupled modes, which cannot be resolved correctly using a single reference curve fitting method. If there are two repeated roots, then at least two references, or responses (rows or columns) of the FRF matrix must be used in order to identify the modal parameters. The same case is with this plate where two modes the third longitudinal bending and torsion are closely coupled.

Twenty-seven measurement points were chosen. Two of them were reference points where two accelerometers were placed in the corners of the shorter side of the plate (points

1 and 19). Fig.5 shows the arrangement of the accelerometers and measurement points. The whole plate was laid on a soft ribbed sponge (the same results would be obtained if it was laid on springs), so it could be said it was tested in a free-free condition. Plate dimensions are shown in Fig.1. Plate mass with accelerometers is 0.936 kg (mass of one accelerometer is 5 g and 8 g with the equipment).

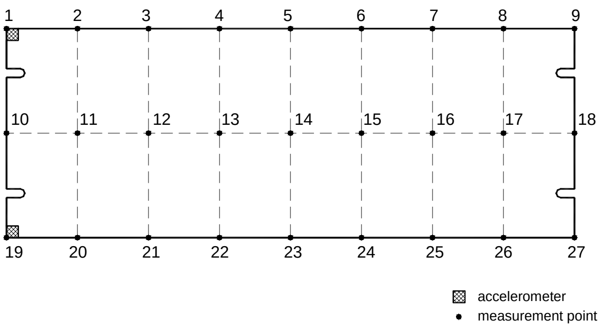

Figure 5. The arrangement of accelerometers and measurement points

Description of the experiment

The structure was impacted at 27 points by roving hammer in $-z$ direction. Two different reference points were used, points 1 and 19. Acceleration was measured in those points in $+z$ directions. As a result, a set of frequency response functions (FRFs) were obtained. Based on them natural frequencies and damping, as well as appropriate mode shapes of the plate, were obtained.

The measurements were made over a frequency range from 0 to 800 Hz with the 0.5 Hz resolution between frequency lines. The measuring range for the accelerometers was ±2236 m/s2 with exponential type of window. The measuring range for the hammer was ±223.6 N with transient type of window. The record time was two seconds for each signal and total number of samples was 2048 in each pass. Linear scale was used and three averages for each point in which the hammer impacted.

The experimental results

Frequency response function – FRF

Fifty-four frequency response functions were obtained (twenty-seven for each accelerometer) and they are shown in Fig.6. The ordinate represents magnitude in linear scale in ((m/s2)/N) and the abscise represents frequency in (Hz) in the range from 0 to 800 Hz.

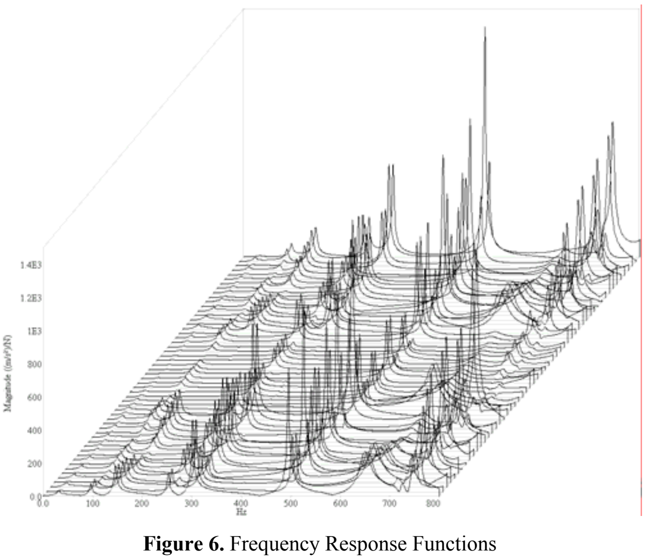

Figure 6. Frequency Response Functions

Natural frequencies, damping and mode shapes

Fig.7 shows all FRFs in the frequency range from 450Hz to 550 Hz and all peaks are around 495 Hz. Each FRF has only one peak around 495 Hz.

Since the results of finite element analysis had given closely coupled modes, it was expected to have them in the experiment too. Therefore, multiple reference modal parameter estimation algorithms should be used for this set of FRFs. For clear presentation, two frequency response functions are given parallel (impulse in point 4 and response in points 1 and 19) in Fig.8. Left cursor is placed at 494 Hz (maximum at lower FRF) and right is placed at 495 Hz (maximum at upper FRF) to see that these two frequencies are so close that they are not visibly separated on the FRFs. Frequency range from 490 Hz to 500 Hz was chosen to magnify peaks of modes. Magnitude is in linear scale.

In the first test case, experimental modal analysis with single reference method extracted two close modes, but adequate mode shapes had the same form. That was the reason why polyreference method the complex mode indicator function (CMIF) in ME'scopeVES was used to find these closely coupled modes in the second test case. Natural frequencies, damping and mode shapes using the previously obtained set of FRFs were obtained by the CMIF. The CMIF plot detected ten modes. All were single modes (not repeated) except the sixth and the seventh mode, which are 1 Hz apart, at 494 Hz and 495 Hz and were closely coupled.

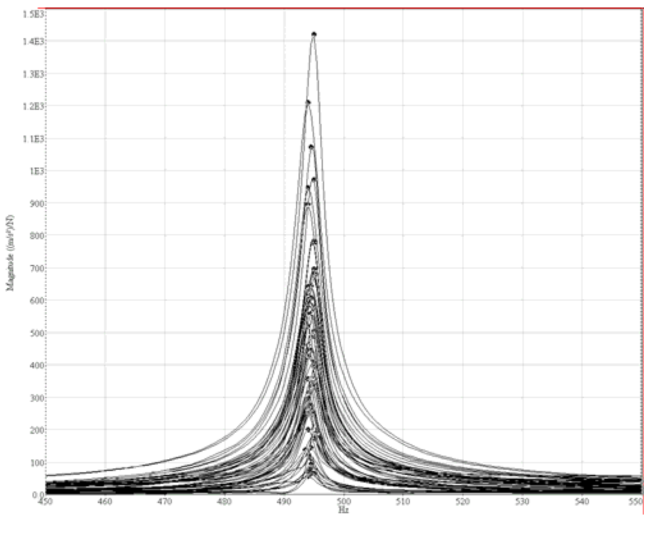

Figure 7. Peaks of frequency response functions around 495 Hz

Frequency and damping results obtained by EMA are shown in Table 2. At the same table, the frequency results by FEA are also given. Comparing the values of natural frequencies from finite element analysis and experimental modal analysis it can be concluded that they are in the permitted area. The only difference occurs in the rigid body mode whose natural frequency is 32.6 Hz obtained experimentally and the result of FEA is 0 Hz. The frequency is different from zero because the plate was laid on a soft support, the ribbed sponge, so the rigid body mode is the mode of the ribbed sponge here. Using the CMIF, the closely coupled modes were successfully extracted in the second test case. Mode shapes - the third longitudinal bending (Fig.9) and the third longitudinal torsion (Fig.10) obtained by EMA have a high degree of similarity with the corresponding mode shapes - third longitudinal bending (Fig.3) and the third longitudinal torsion (Fig.4) obtained by FEA.

The same procedure was done in software I-DEAS also by polyreference method and the results were the same.

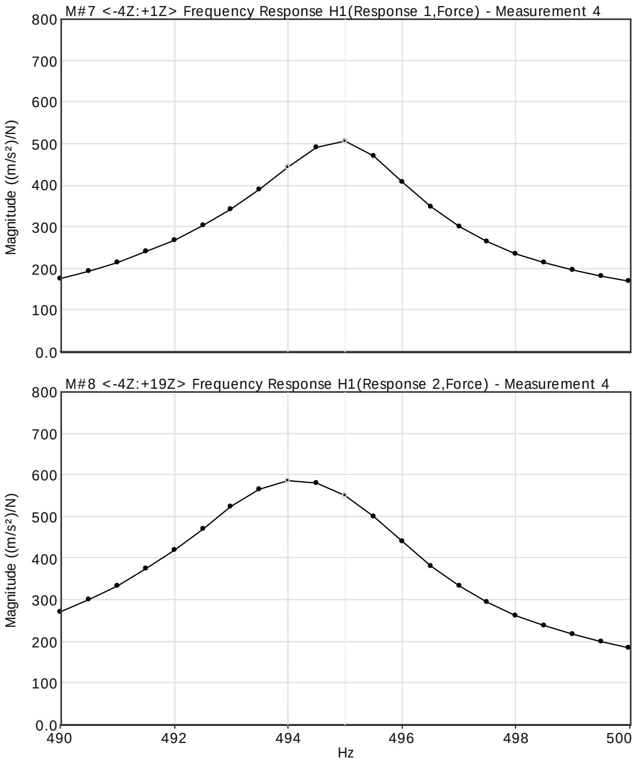

Figure 8. Two FRFs in frequency range from 490 Hz to 500 Hz (impulse in point 4 and response in points 1 and 19)

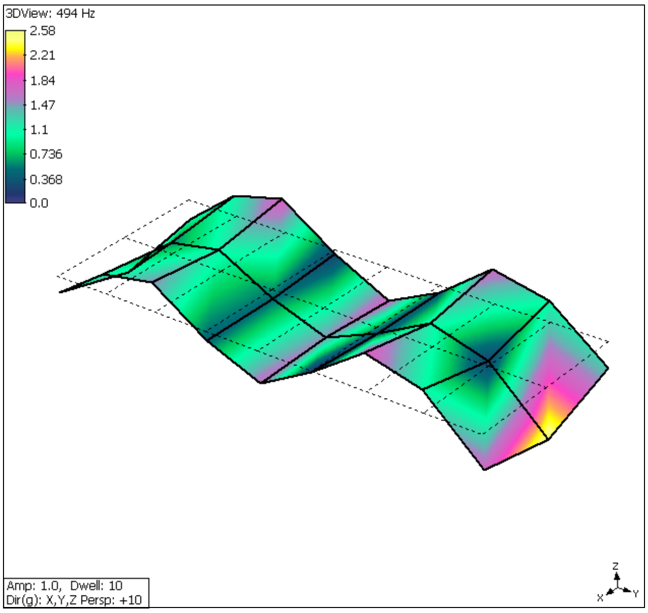

Figure 9. The third longitudinal bending

Table 2. The results of EMA and FEA

	Description	EMA		FEA
1.	Frequency of the ribbed sponge	32.6	9.68	-
2.	Frequency of the first longitudinal bending	96.4	2.75	89.72
3.	Frequency of the first longitudinal torsion	145	1.91	143.22
4.	Frequency of the second longitudinal bending	253	0.727	250.85
5.	Frequency of the second longitudinal torsion	301	0.562	303.35
6.	Frequency of the third longitudinal bending	494	0.3	496.11
7.	Frequency of the third longitudinal torsion	495	0.3	501.61
8.	Frequency of the first transversal bending	648	1.64	655.90
9.	Frequency of the second transversal bending	723	0.46	729.72
10.	Frequency of the fourth longitudinal torsion	745	0.389	758.96

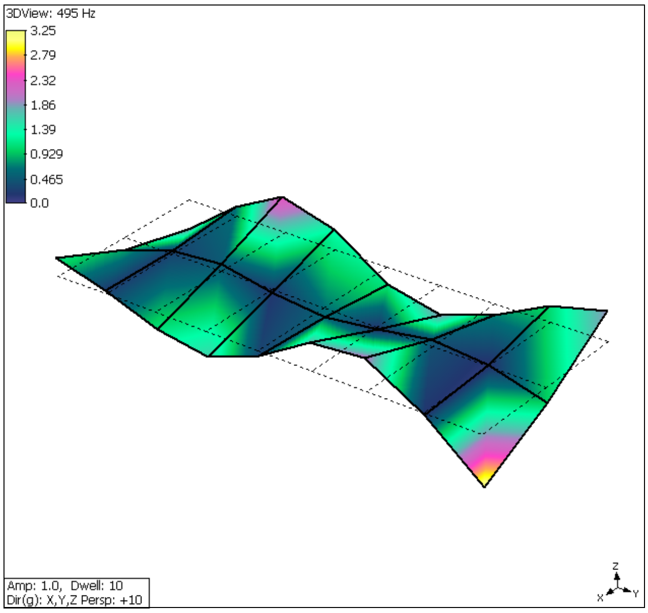

Figure 10. The third longitudinal torsion

Conclusion

In this paper, complex mode indicator function was applied to find and estimate modal parameters of closely coupled modes. The results of finite element analysis and experimental modal analysis were also compared. One rectangular plate was chosen as an example for demonstrating the extraction of closely coupled modes. Software I-DEAS was used for finite element analysis. The finite element model of the plate was generated and normal mode dynamic analysis by Lanczos method was applied to it. Two closely coupled modes were found - the third longitudinal bending and the third longitudinal torsion. The natural frequency results obtained by FEA are presented in Table 1 and mode shapes of close coupled modes are shown in Figures 3 and 4.

After FEA, experimental modal analysis was applied to the plate. Two reference points (where the accelerometers were mounted) and 27 measurement points (where the hammer impacted) were chosen. As a result, a set of frequency response functions (FRFs) was obtained. The plate model and its set of FRFs were then exported to ME'scopeVES where those FRFs were processed using the CMIF method. The CMIF method recovered the modal parameters of all the modes including closely coupled modes - the third longitudinal bending and the third longitudinal torsion. Frequency and damping values obtained by EMA are presented in Tabele 2 and mode shapes are shown in Fig.9 – Fig.10. Comparing the values of all natural frequencies, obtained by FEA and EMA (Table 2) it was concluded that the results are in the permitted area. In addition, on comparing the mode shapes of closely coupled modes obtained by FEA and EMA, a high degree of similarity was found.

Finally, it can be concluded that the CMIF method successfully separated closely coupled modes. The purpose of this paper was to overwhelm the methods for finding and estimating the modal parameters of closely coupled modes represented on a simple object such as the rectangular plate.

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Primena funkcije za indikaciju kompleksnih modova pri izdvajanju blisko spregnutih modova, ili modova sa višestrukim korenima

Metoda konačnih elemenata u okviru Numeričke modalne analize i Eksperimentalna modalna analiza sa više referentnih tačaka su korišćene da bi se našli i ocenili modalni parametri blisko spregnutih modova na odabranoj strukturi, pravougaonoj aluminijumskoj ploči. Ploča je konstruisana tako da ima blisko spregnute modove. Cilj je bio izdvajanje ovakvih modova na ploči koristeći skup funkcija frekventnog odziva koje su dobijene kao rezultat Eksperimentalne modalne analize. Estimacija modalnih parametara ploče je obavljena uz pomoć Funkcije za indikaciju kompleksnih modova u *ME'scopeVES*-u. Metoda konačnih elemenata je primenjena da bi se utvrdilo postojanje blisko spregnutih modova pre primene Eksperimentalne modalne analize.

Ključne reči: **polireferentna metoda, modalna analiza, metoda konačnih elemenata, funkcije frekventnog odziva, *Funkcija za indikaciju kompleksnih modova*, bliski modovi, oblik oscilovanja, prigušenje.**

Применение функции для индикации комплексных режимов при выделении близкосопряжённых режимов или режимов со многократными корнями

Метод конечных элементов в рамках численного модального анализа и экспериментальный модальный анализ со множеством расчётных точек использованы чтобы было возможно найти и оценить модальные параметры близкосопряжённых режимов на выбраной структуре, на прямоугольной алюминиевой плите. Конструкция плиты такова, что у неё близкосопряжённые режимы. Это сделано со целью выделения таких режимов на плите пользуя совокупность функций частотной чувствительности, которые получены в форме итога экспериментального модального анализа. Оценка модальных параметров плиты проведена при помощи функции для индикации комплексных режимов в ***ME'scopeVES*-u**. Метод конечных элементов применён чтобы было установлено существование близкосопряжённых режимов перед применением экспериментального модального анализа.

Ключевые слова: мультипунктный метод, анализ режимов работы (модальный метод), метод конечных элементов, функции частотной чувствительности, функция для индикации комплексных режимов, близкие режимы, частота, форма колебания, демпфирование.

Application de la fonction pour indiquer les modes complexes lors de la séparation des modes couplés de près ou des modes à multiples racines

La méthode des éléments finis, dans le cadre de l'analyse numérique modale et l'analyse expérimentale modèle aves plusieurs points référentielles ont été utilisées afin de trouver et évaluer les paramètres modaux des modes couplés de près pour la structure choisie, une plaque rectangulaire en alluminium. La plaque est construite de façon à posséder les modes couplés de près. Le but était de séparer ces modes sur la plaque en utilisant l'ensemble des fonctions de réponse fréquente obtenues comme résultat de l'analyse expérimentale modale. L'estimation des paramètres modaux de la plaque a été faite à l'aide de fonction pour l'indication des modes complexes en ME'scope VES.La méthode des éléments finis était appliquée pour déterminer l'existence des modes couplés de près avant l'application d'analyse expérimentale modale.

Mots clés: **méthode polyréférentielle, analyse modale, méthode des éléments finis, fonctions de réponse fréquente, fonction pour indiquer les modes complexes, modes couplés de près, forme d'oscillation, étouffage.**